

函数模型在概率中的应用第 2 课时拓展提升任务答案

一般地, 若离散型随机变量 X 的分布列为

X	x_1	x_2	\cdots	x_i	\cdots	x_n
P	p_1	p_2	\cdots	p_i	\cdots	p_n

求证: (1) $E(aX + b) = aE(X) + b$;

$$(2) D(aX + b) = a^2D(X)$$

解析:

$$\begin{aligned} E(aX + b) &= (ax_1 + b)p_1 + (ax_2 + b)p_2 + \cdots + (ax_n + b)p_n \\ &= a(x_1p_1 + x_2p_2 + \cdots + x_np_n) + b(p_1 + p_2 + \cdots + p_n) \\ &= aE(X) + b \end{aligned}$$

$$\begin{aligned} D(aX + b) &= [(ax_1 + b) - (aE(X) + b)]^2 p_1 + [(ax_2 + b) - (aE(X) + b)]^2 p_2 \\ &\quad + \cdots + [(ax_n + b) - (aE(X) + b)]^2 p_n \\ &= a^2[(x_1 - E(X))^2 p_1 + (x_2 - E(X))^2 p_2 + \cdots + (x_n - E(X))^2 p_n] \\ &= a^2D(X) \end{aligned}$$