

复数的乘、除运算学习任务单答案

例 1 (1) $(3+4i)(-2-3i)$ (2) $(1-2i)(3+4i)(-2+i)$

$$\begin{aligned} &= -6 - 8i - 9i - 12i^2 \\ &= 6 - 17i \end{aligned} \qquad \begin{aligned} &= (11-2i)(-2+i) \\ &= -20 + 15i \end{aligned}$$

例 2 (1) $(1+i)^2$ (2) $(2+3i)(2-3i)$

$$\begin{aligned} &= 1 + 2i + i^2 \\ &= 1 + 2i - 1 \\ &= 2i \end{aligned} \qquad \begin{aligned} &= 2^2 - (3i)^2 \\ &= 4 - (-9) \\ &= 13 \end{aligned}$$

※填空① $(1-i)^2 = \underline{-2i}$

② $i^1 = \underline{i}$; $i^2 = \underline{-1}$; $i^3 = \underline{-i}$; $i^4 = \underline{1}$; $i^5 = \underline{i}$;

你发现什么规律了吗? 你能直接完成下面这个填空吗?

$i^{4n+1} = \underline{i}$;

例 3: 计算

$$\begin{aligned} (1) \quad (1+i) \div (1-i) &= \frac{1+i}{1-i} \\ &= \frac{(1+i)(1+i)}{(1-i)(1+i)} = \frac{1+2i+i^2}{1-i^2} \\ &= \frac{2i}{2} = i \end{aligned}$$

$$\begin{aligned} (2) \quad \frac{(-1+i)(2+i)}{-i} &= \frac{-2+2i-i+i^2}{-i} \\ &= \frac{(i-3)i}{-i \cdot i} \\ &= -1-3i \end{aligned}$$

例 4: 在复数范围内解下列方程

(1) $9x^2 + 16 = 0$

$$(1) \quad 9x^2 + 16 = 0 \Rightarrow x^2 = -\frac{16}{9} = \frac{16}{9}i^2 = \left(\frac{4}{3}i\right)^2$$

$$\text{故 } x = \pm \frac{4}{3}i$$

(2) $ax^2 + bx + c = 0$, 其中 $a, b, c \in R, a \neq 0$

(2) $ax^2 + bx + c = 0 \Rightarrow (x + \frac{b}{2a})^2 = \frac{b^2 - 4ac}{4a^2}$

(i) 当 $\Delta = b^2 - 4ac > 0$ 时, $x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$;

(ii) 当 $\Delta = b^2 - 4ac < 0$ 时, 因为 $-\Delta = 4ac - b^2 > 0$, 所以 $x = -\frac{b}{2a} \pm \frac{\sqrt{4ac - b^2}}{2a}i$