

参考答案:

1. $\frac{2}{3}$

2. $-\frac{4}{5}$

3 解: $\because \cos(\alpha + \beta) = \frac{4}{5}, \alpha + \beta \in (\frac{3}{2}\pi, 2\pi)$

$$\therefore \sin(\alpha + \beta) = -\frac{3}{5}$$

又 $\because \cos(\alpha - \beta) = -\frac{4}{5}, \alpha - \beta \in (\frac{\pi}{2}, \pi)$

$$\therefore \sin(\alpha - \beta) = \frac{3}{5}$$

$$\therefore \cos 2\beta = \cos[(\alpha + \beta) - (\alpha - \beta)]$$

$$= \cos(\alpha + \beta)\cos(\alpha - \beta) + \sin(\alpha + \beta)\sin(\alpha - \beta)$$

$$= \frac{4}{5} \times (-\frac{4}{5}) + (-\frac{3}{5}) \times \frac{3}{5} = -\frac{16}{25} - \frac{9}{25} = -1$$

4. 解 (I) 因为 $\pi < \alpha < \frac{3\pi}{2}$, $\sin \alpha = -\frac{4}{5}$,

故 $\cos \alpha = -\sqrt{1 - \sin^2 \alpha} = -\frac{3}{5}$.

$$(II) \sin 2\alpha + 3\tan \alpha = 2\sin \alpha \cos \alpha + 3 \times \frac{\sin \alpha}{\cos \alpha} = 2 \times \left(-\frac{4}{5}\right) \times \left(-\frac{3}{5}\right) + 3 \times \frac{\left(-\frac{4}{5}\right)}{\left(-\frac{3}{5}\right)} = 4\frac{24}{25}.$$

5. 解: (1): $\because \cos(x - \frac{\pi}{4}) = \frac{\sqrt{2}}{10}$, 又 $x \in (\frac{\pi}{2}, \frac{3\pi}{4})$

$$\therefore x - \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\therefore \sin\left(x - \frac{\pi}{4}\right) = \frac{7\sqrt{2}}{10}$$

$$\begin{aligned}
 \therefore \sin x &= \sin\left(x - \frac{\pi}{4} + \frac{\pi}{4}\right) \\
 &= \sin\left(x - \frac{\pi}{4}\right) \cos \frac{\pi}{4} + \cos\left(x - \frac{\pi}{4}\right) \sin \frac{\pi}{4} \\
 &= \frac{7\sqrt{2}}{10} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{10} \times \frac{\sqrt{2}}{2} = \frac{4}{5}
 \end{aligned}$$

(2): 由 (1) 知 $\sin x = \frac{4}{5}$, $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$$\therefore \cos x = -\frac{3}{5}$$

$$\therefore \sin 2x = -\frac{24}{25}, \cos 2x = -\frac{7}{25}$$

$$\begin{aligned}
 \therefore \sin\left(2x + \frac{\pi}{3}\right) &= \sin 2x \cos \frac{\pi}{3} + \cos 2x \sin \frac{\pi}{3} \\
 &= \left(-\frac{24}{25}\right) \times \frac{1}{2} - \frac{7}{25} \times \frac{\sqrt{3}}{2} = -\frac{24 + 7\sqrt{3}}{50}
 \end{aligned}$$