

第十五讲 三角恒等变换精讲—拓展提升

1. $\frac{14}{5}$.

2. $-\frac{3}{5}, \frac{\sqrt{2}}{10}$.

3. $[4k\pi - \frac{13\pi}{6}, 4k\pi - \frac{\pi}{6}] (k \in \mathbb{Z})$

4. $\pi; \left(0, \frac{\pi}{6}\right]$ 或 $\left(0, \frac{\pi}{6}\right), \frac{3}{2}$

5. 解: (1): $\because \cos(x - \frac{\pi}{4}) = \frac{\sqrt{2}}{10}$, 又 $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$$\therefore x - \frac{\pi}{4} \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$$

$$\therefore \sin\left(x - \frac{\pi}{4}\right) = \frac{7\sqrt{2}}{10}$$

$$\therefore \sin x = \sin\left(x - \frac{\pi}{4} + \frac{\pi}{4}\right)$$

$$= \sin\left(x - \frac{\pi}{4}\right) \cos \frac{\pi}{4} + \cos\left(x - \frac{\pi}{4}\right) \sin \frac{\pi}{4}$$

$$= \frac{7\sqrt{2}}{10} \times \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{10} \times \frac{\sqrt{2}}{2} = \frac{4}{5}$$

(2): 由 (1) 知 $\sin x = \frac{4}{5}$, $x \in \left(\frac{\pi}{2}, \frac{3\pi}{4}\right)$

$$\therefore \cos x = -\frac{3}{5}$$

$$\therefore \sin 2x = -\frac{24}{25}, \cos 2x = -\frac{7}{25}$$

$$\begin{aligned} \therefore \sin\left(2x + \frac{\pi}{3}\right) &= \sin 2x \cos \frac{\pi}{3} + \cos 2x \sin \frac{\pi}{3} \\ &= \left(-\frac{24}{25}\right) \times \frac{1}{2} - \frac{7}{25} \times \frac{\sqrt{3}}{2} = -\frac{24 + 7\sqrt{3}}{50} \end{aligned}$$